Centre for Doctoral Training
Mathematics of Random Systems

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University of Oxford
Mathematical Institute

Monday 24 April	
Analysis: Stochastic processes, SPDEs, Random Graphs	
9:20-9:50	Registration
9:50-10:00	Welcome
10:00-10:50	**Keynote lecture** **Ilya Chevyrev (Edinburgh University)** *Stochastic analysis in constructive field theory*
Coffee break	
11:20-11:50	**Benedikt Petko** Coarse curvature of weighted Riemannian manifolds with application to random geometric graphs
11:50-12:20	**Zheneng Xie** Directed random graphs and queues
12:20-12:50	**Laszlo Mikolas** On the continuity equation on co-evolving graphs
Lunch	
14:00-14:30	**Julian Sieber** On the (non-)stationary density of fractional SDEs
14:30-15:00	**Julian Meier** Particle systems on the positive half-line with boundary interactions
15:00-15:30	**Harprit Singh** Singular equations on homogeneous Lie groups
Coffee break	
16:00-16:50	**Keynote lecture** **Ellen Powell (Durham University)** *Brownian excursions, conformal loop ensembles and critical Liouville quantum gravity*
19:00	Conference dinner

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Tuesday 25 April	
Modelling: Stochastic and data driven finance, mathematical biology	
09:00-09:30	Tea & Coffee
9:30-10:20	**Keynote lecture** **Eyal Neuman (Imperial College London)** *New mathematical challenges arising from propagator models*
10:20-10:50	**Felix Prenzel** Simulation of limit order books
Coffee break	
11:20-11:50	**Zan Zuric** The grey Bergomi model
11:50-12:20	**Lancelot Da Costa** From interacting stochastic dynamics to models of cognition and decision-making
12:20-12:50	**Yihuang Zhang** Random vortex methods for the Navier—Stokes equation
Lunch	
14:00-16:30	Excursion
Coffee	
17:00-17:30	**Alain Rossier** Deep learning, an asymptotic viewpoint
18:00-19:00	Public lecture **Des Higham (Edinburgh University)** *Deep learning: What could go wrong?*
19:00-20:00	Drinks reception

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Wednesday 26 April	
Algorithms: Machine learning, stochastic simulation, optimal control	
9:00	Tea & coffee
9:30-10:00	**Jonathan Tam** Markov decision processes with controllable observations
10:00-10:30	**Deven Sethi (Edinburgh University)** Gradient flow arising from the modified MSA for stochastic control problems
10:30-11:00	**Akshunna S Dogra** Shaping up scientific machine learning
Coffee break	
11:30-12:00	**Verena Schwarz (University of Klagenfurt)** Approximation and optimality for jump-diffusion SDEs with discontinuous drift
12:00-12:30	**Xingyuan Chen (Edinburgh University)** A study on the super-linear McKean—Vlasov SDEs and associated particle systems
Lunch	
13:30-14:20	Keynote lecture **Sam Cohen (University of Oxford)** Neural Q-learning solutions to elliptic PDEs
14:20-14:30	Closing remarks

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Day 1, Monday 24 April

Analysis: Stochastic processes, SPDEs, Random Graphs

10:00 Keynote Talk: Ilya Chevyrev (Edinburgh)

Stochastic analysis in constructive field theory

Recent years have seen many new ideas appearing in the solution theories of singular stochastic partial differential equations. An exciting application of SPDEs that is beginning to emerge is to the construction and analysis of quantum field theories. In this talk, I will describe how stochastic quantisation of Parisi-Wu can be used to study QFTs, especially those arising from gauge theories, the rigorous construction of which is largely open.

11:20 CDT Student Talk: Benedikt Petko (Imperial College London)

Coarse curvature of weighted Riemannian manifolds with application to random geometric graphs

Prof Marc Arnaudon, Prof Xue-Mei Li

Coarse curvature is an alternative approach to classical Ricci curvature of Riemannian manifolds and is based on the expansion of the Wasserstein distance of probability distributions on the manifold. This yields a notion of curvature that is well-defined also on non-smooth spaces, in particular graphs.

In our work, we showed that coarse curvature is consistent as well in the presence of a potential on the manifold, which corresponds in a unique way to a weighting reference measure. In this case, we retrieve the generalized Ricci tensor, with adjustment by the Hessian of the potential. We present an application of our finding in showing that the coarse curvature of a random graph sampled from a weighted Riemannian manifold converges to the classical generalized Ricci curvature tensor as the sample size increases.

11:50 CDT Student Talk: Zheneng Xie (University of Oxford)

Directed Random Graphs and Queues

Scaling limits of undirected graphs have been studied in great detail. However, most real life networks have directionality, such as web links, financial transactions or disease transmission. In this talk we will present scaling limits for classes of directed scaling models at criticality. We will also present how certain random graphs can be obtained from a queueing process and how this can assist in proving results.

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12:20 Selected Abstract Laszlo Mikolas (University of Oxford)

On the continuity equation on co-evolving graphs

In this talk we consider a class of non-local continuity equations on co-evolving graphs, i.e. the underlying graph evolves in time and its dynamics depends on the solution of the continuity equation. This type of models have recently received wide attention for their applications to, e.g.,opinion dynamics and data science.

We study their well-posedeness, the behaviour at different time scales, and the discrete-to-continuum problem.

1400 CDT Student Talk: Julian Sieber (Imperial College London)

On the (Non-)stationary density of fractional SDEs

I will present a novel approach for studying the density of SDEs driven by additive fractional Brownian motion. It allows us to establish smoothness and Gaussian-type upper and lower bounds for both the non-stationary as well as the stationary density. While the stationary density has not been studied in any previous works, the former was the subject of multiple articles by Baudoin, Hairer, Nualart, Ouyang, Pillai, Tindel, among others. The common them of all of these works is to obtain the results through bounds on the Malliavin derivative. The main disadvantage of this approach lies in the non-optimal regularity conditions on the SDE's coefficients. In case of additive noise, the equation is known to be well-posed if the drift is merely sublinear and measurable (resp. Holder continuous). Relying entirely on classical methods of stochastic analysis (avoiding any Malliavin calculus), we prove the aforementioned Gaussian-type bounds under optimal regularity conditions.

The talk is based on a joint work with Xue-Mei Li and Fabien Panloup.

14:30 CDT Student Talk: Julian Meier (University of Oxford)

Particle Systems on the Positive Half-Line with Boundary Interactions

We study particle systems on the positive half-line with interaction with the boundary at zero. We show weak convergence of particle systems with elastic and reflecting boundaries to the unique solutions of related limit SPDEs. The interaction of the particles is given through a common noise which leads to linear limit SPDEs. Moreover, we show existence of densities and integrability away from the boundary for these systems. We also consider systems with a sticky boundary. We show how the sticky boundary leads to different regularity results at the boundary. A third type of particle models we consider have a stronger interaction with the boundary and experience feedback through elastic stopping times. We show weak convergence of this particle system to the solution of a McKean-Vlasos equation. In order to establish this weak convergence we prove uniqueness of solutions of the McKean-Vlasov equation given suitably defined solution concepts that respect the discontinuities that can appear in these systems.

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1530 CDT Student Talk: Harprit Singh (Imperial College London)

Singular Equations on homogeneous Lie Groups

Co-Authors: Avi Mayorcas (TU Berlin)

We show that Regularity Structures, without much change, work on general homogeneous Lie groups. In particular we obtain a solution theory for SPDEs where the differential operator stems from a large class of hypo-elliptic operators.

1600 Keynote Talk: Ellen Powell (Durham University)

Brownian excursions, conformal loop ensembles and critical Liouville quantum gravity

It was recently shown by Aïdékon and Da Silva how to construct a growth fragmentation process from a planar Brownian excursion. I will explain how this same growth fragmentation process arises in another setting: when one decorates a certain “critical Liouville quantum gravity random surface” with a conformal loop ensemble of parameter 4.

This talk is based on joint work with Juhan Aru, Nina Holden and Xin Sun.

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Tuesday 25 April

Modelling: Stochastic and data driven finance, mathematical biology

0930 Keynote Talk: Eyal Neuman (Imperial College London)

New Mathematical Challenges Arising from Propagator Models

Price impact refers to the empirical fact that execution of a large order affects the risky asset's price in an adverse and persistent manner leading to less favourable prices. Propagator models help us to quantity the price impact. They express price moves in terms of the influence of past trades, convoluted with a price impact kernel function. We consider a class of optimal liquidation problems where the agent's transactions create transient price impact driven by a Volterra-type propagator along with temporary price impact. We formulate these problems as minimization of a revenue-risk functionals, where the agent also exploits available information on a progressively measurable price predicting signal. By using an infinite dimensional stochastic control approach, we derive analytic solutions to these equations which yields an explicit expression for the optimal trading strategy.

We then consider a class of learning problems in which an agent liquidates a risky asset while creating transient price impact driven by an unknown propagator. We characterize the trader's performance as maximization of a revenue-risk functional, where the trader also exploits available information on a price predicting signal. We present a trading algorithm that alternates between exploration and exploitation phases and achieves sublinear regrets with high probability. For the exploration phase we propose a novel approach for non-parametric estimation of the price impact kernel by observing only the visible price process and derive sharp bounds on the convergence rate, which are characterised by the singularity of the propagator.

The talk is based on joint projects with Eduardo Abi-Jaber and Yufei Zhang

10:20 CDT Student Talk: Felix Prenzel (University of Oxford)

Simulation of limit order books

For many applications, market participants are more interested in time series of limit order book (LOB) snapshots rather than the precise event stream. One reason for this is sufficient amount of information in the evolution of such snapshots.

This study leverages generative adversarial networks (GANs) to model and simulate time series of LOB snapshot. More precisely, we learn the conditional probability distribution of future LOB snapshots based on past states. Our results show GANs are able to learn LOB transitions and capture a variety of stylized facts when properly tuned. Recurrent simulation allows to generate entire time series of order book data.

Finally, we show that the trained generator can reflect stylized properties from market impact studies. In particular, the model shows a decaying marginal impact of trade size, higher impact

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of liquidity extraction than liquidity provision as well as decreasing impact with increasing depths. These results are even more interesting as the model learns these properties despite not being trained to do so and a very reduced state space. Our research opens plenty of avenues which can be pursued in this direction. These are the inclusion of longer history, more state variables as well as an improved training procedure amongst others.

11:20 CDT Student Talk: Zan Zuric (Imperial College London)

The grey Bergomi model

In this talk, we will present our work in progress, where we discuss a possible candidate model for a joint SPX and VIX calibration based on the so-called grey Brownian motion, generalisation of fractional Brownian motion with a random diffusivity parameter. In general, the process is non-Gaussian, which makes it a more realistic model for capturing the distributional properties of VIX and generates a positive skew of the VIX smile.

This is a joint work in progress with Antoine Jacquier.

11:50 CDT Student Talk: Lancelot Da Costa (Imperial College London)

From interacting stochastic dynamics to models of cognition and decision-making

We consider coupled stochastic processes at non-equilibrium steady state, which describe the interaction between external, internal, sensory and active states of a particle or agent. We unpack the consequences of this in terms of approximate Bayesian inference, in the sense that internal states can be seen as continuously inferring external states, consistently with variational inference in Bayesian statistics and theoretical neuroscience. Beyond this, we single out an interesting sort of non-equilibrium steady-state reminiscent of biological systems, wherein the states of a particle have low entropy fluctuations. Their trajectories can be governed by a variational principle of least action over a functional relating to optimal Bayesian design and decision-making. We then use this functional to simulate eye and Human arm movements, and solve reinforcement learning tasks, showcasing its usefulness as a model of intelligent biological behaviour.

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16:30 CDT Student Talk: Alain Rossier (University of Oxford)

Deep Learning, an asymptotic viewpoint

Neural networks have improved the state-of-the-art on a wide range of complex high-dimensional learning problems. However, the theoretical framework of classical learning theory fails to explain the power of neural networks. The empirical observations regarding the convergence of the optimization algorithm despite the non-convexity of the loss function, the generalization capacity of overparametrized networks, and the role of depth, lack a solid mathematical theory. We aim to shed light on these timely questions by decomposing the out-of-sample error into three terms: approximation, optimization, and generalization error. For neural networks - unlike traditional statistical models - these quantities feed on each other in non-trivial ways. We seek to give partial answers to these questions by investigating asymptotic limits, such as when the width or the depth of the network tends to infinity, when the training time step approaches zero, or when the number of data points tends to infinity.

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<u>1800 Public Lecture: Des Higham (University of Edinburgh)</u>

Deep Learning: What Could Go Wrong?

A traffic "Stop" sign on the roadside can be misinterpreted by a driverless vehicle as a speed limit sign when minimal graffiti is added. Wearing a pair of adversarial spectacles can fool facial recognition software into thinking that we are Brad Pitt. The vulnerability of artificial intelligence (AI) systems to such adversarial interventions raises questions around security and ethics, and many governments are now considering proposals for their regulation. I believe that mathematicians can contribute to this landscape. We can certainly get involved in the conflict escalation issue, where new defence strategies are needed to counter an increasingly sophisticated range of attacks. Perhaps more importantly, we also have the tools to address big picture questions, such as: What is the trade-off between robustness and accuracy? Can any AI system be fooled? Do proposed regulations make sense? Focussing on deep learning algorithms, I will describe how mathematical concepts can help us to understand and, where possible, ameliorate current limitations in AI technology.

Des Higham is a Professor of Numerical Analysis in the School of Mathematics at the University of Edinburgh. He has research interests in the design and evaluation of computational methods, and their applications in network science, data analytics and machine learning. He is a Fellow of the Royal Society of Edinburgh, of the Alan Turing Institute, and of the Society for Industrial and Applied Mathematics (SIAM). He is Editor-in-Chief of the journal SIAM Review and recently held an Established Career Fellowship from UK Research and Innovation. In 2020 he was awarded a Shephard Prize from the London Mathematical Society for research making a contribution to mathematics with a strong intuitive component which can be explained to those with little or no knowledge of university mathematics.

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Day 3, Wednesday 26 April

Algorithms: Machine learning, stochastic simulation, optimal control

09:30 CDT Student Talk: Jonathan Tam (University of Oxford)

Markov Decision Processes with Controllable Observations

Markov Decision Processes is a common mathematical framework employed in decision making to factor in uncertainty during the optimisation process. We explore variations to the standard setup which involve actively controlled observations. Exploration through the control of the quality and accuracy of observations is combined with the exploitation of the gathered information to maximise expected rewards. This differs from standard partial information setups, where one has access to a stream of noisy information which is not subject to active controls. The trade-off is akin to the exploration vs exploitation problem in reinforcement learning. We demonstrate this idea of controlled observations in two frameworks, one that stipulates a payment for observing at each instant, and another on shortening information delay with a premium. We show that the value functions satisfy a system of quasi-variational inequalities, and we discuss aspects of the numerical implementation with some numerical examples.

10:00 Selected Abstract: Deven Sethi (University of Edinburgh)

Gradient Flow arising from the Modified MSA for Stochastic Control problems.

Dr David Siska - University of Edinburgh

The modified Method of Successive Approximations (MSA) is an iterative scheme for approximating solutions to stochastic control problems in continuous time based on Pontryagin Optimality Principle which, starting with an initial open loop control, solves the forward equation, the backward adjoint equation and then performs a static minimization step. We observe that this is an implicit Euler scheme for a gradient flow system. We prove that appropriate interpolations of the iterates of the modified MSA converge to a gradient flow. We then study the convergence of this gradient flow as time goes to infinity. In the general (non-convex) case we prove that the gradient term itself converges to zero. This is a consequence of an energy identity we establish which shows that the optimization objective decreases along the gradient flow. Moreover, in the convex case, when Pontryagin Optimality Principle provides a sufficient condition for optimality, we prove that the optimization objective converges at rate 1/S to its optimal value. The main technical difficulties lie in obtaining appropriate properties of the Hamiltonian (growth, continuity). These are obtained by utilising the theory of Bounded Mean Oscillation (BMO) martingales required for estimates on the adjoint Backward Stochastic Differential Equation (BSDE).

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10:30 Selected Abstract: Akshunna S. Dogra (Imperial College London)

Shaping up scientific Machine Learning

Prof. Demba Ba (Harvard University), Mr. Rikab Gambhir (MIT) Prof. Abiy Tasissa (Tufts University), Prof. Jesse Thaler (MIT)

The identification of interesting substructures within jets is an important tool for searching for new physics beyond the Standard Model at colliders. Many of these substructure tools have previously been shown to take the form of optimal transport problems, in particular the Energy Mover's Distance (EMD). In this work, we show that the EMD is in fact the natural structure for comparing collider events, which accounts for its success in understanding event and jet substructure over the last decade. We then present a Shape Hunting Algorithm using Parameterized Energy Reconstruction (SHAPER), which is a general framework for defining and computing shape-based observables. SHAPER generalizes N-jettiness from point clusters to any extended, parametrizable shape. This is accomplished by efficiently minimizing the EMD between events and parameterized manifolds of energy flows representing idealized shapes, implemented using the dual-potential Sinkhorn approximation of the Wasserstein metric. We show how the geometric language of observables as manifolds can be used to define novel observables with built-in infrared-and-collinear safety, a critical physical constraint for our chosen regime of interest. We demonstrate the efficacy of the SHAPER framework by performing empirical jet substructure studies using several examples of new shape-based observables.

11:30 Selected Abstract: Verena Schwarz (University of Klagenfurt)

Approximation and optimality for jump-diffusion SDEs with discontinuous drift

Paweł Przybyłowicz (AGH University of Science and Technology), Alexander Steinicke (Montanuniversität Leoben), Michaela Szölgyenyi (University of Klagenfurt)

In this talk we consider the approximation of jump-diffusion stochastic differential equations with discontinuous drift, possibly degenerate diffusion coefficient, and Lipschitz continuous jump coefficient. We present a jump-adapted higher-order scheme, the so-called transformation-based jump-adapted quasi-Milstein scheme. For this scheme, we provide a complete error analysis: We prove convergence order $3/4$ in L^p for $p\in[1,\infty)$. Further, we provide lower error bounds for non-adaptive and jump-adapted approximation schemes of order $3/4$ in L^1. This result is based on a universal Skorokhod measurable functional representation of solutions to semimartingale SDEs and yields optimality of the transformation-based jump-adapted quasi-Milstein scheme.

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12:00 Selected Abstract: Xingyuan Chen (University of Edinburgh)

A study on the super-linear McKean--Vlasov SDEs and associated particle systems

Goncalo dos Reis (Univeristy of Edinburgh), Wolfgang Stockinger (Imperial College London)

We study a class of McKean--Vlasov Stochastic Differential Equations (MV-SDEs) with drifts and diffusions having super-linear growth in measure and space -- the maps have general polynomial form but also satisfy a certain monotonicity condition. The combination of the drift's super-linear growth in measure (by way of a convolution) and the super-linear growth in space and measure of the diffusion coefficient require novel technical elements in order to obtain the main results. We establish wellposedness, propagation of chaos (PoC), and under further assumptions on the model parameters we show an exponential ergodicity property alongside the existence of an invariant distribution. Further, we present a particle system based Euler-type split-step scheme (SSM) for the simulation of this type of MV-SDEs. We show that the scheme under different assumptions attains different types of strong error around $1/2$. All findings are illustrated by numerical examples with comparison with other possible schemes.

13:30 Keynote Talk: Sam Cohen (University of Oxford)

Neural Q-learning solutions to elliptic PDEs

Based on joint work with Deqing Jiang and Justin Sirignano

Solving high-dimensional partial differential equations (PDEs) is a major challenge in scientific computing. We develop a new numerical method for solving elliptic-type PDEs by adapting the Q-learning algorithm in reinforcement learning. Using a neural tangent kernel (NTK) approach, we prove that the neural network approximator for the PDE solution, trained with the Q-PDE algorithm, converges to the trajectory of an infinite-dimensional ordinary differential equation (ODE) as the number of hidden units becomes infinite. For monotone PDE (i.e. those given by monotone operators), despite the lack of a spectral gap in the NTK, we then prove that the limit neural network, which satisfies the infinite-dimensional ODE, converges in L^2 to the PDE solution as the training time increases. The numerical performance of the Q-PDE algorithm is studied for several elliptic PDEs.